

# Counter-example Guided Abstract Refinement for Verification of Neural Networks

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# Agenda

- Context
- Abstraction algorithms
- Refinement / CEGAR
- Evaluation
- Remarks

# Context

## Neural Networks

Neural Networks (NNs) are widespread and «fashionable»

NNs provide fast classification and regression results

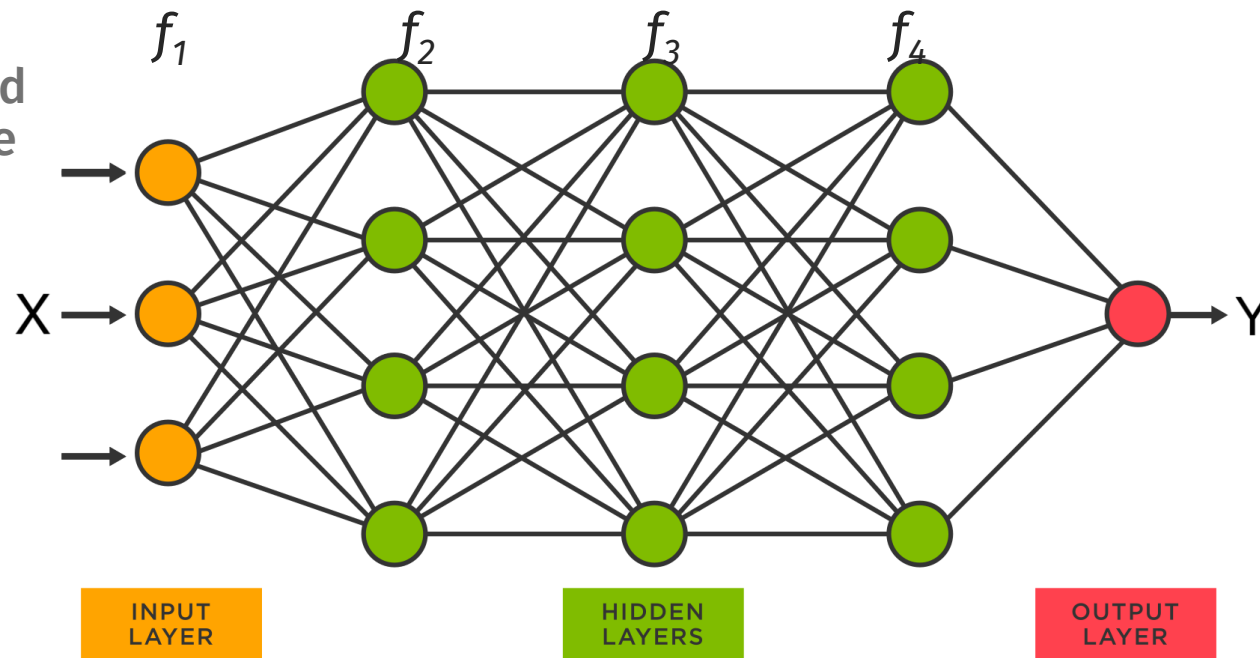
Many success stories in Natural Language Processing, Computer Vision, Control...

# Context

## Neural Networks

Nodes are connected through linear affine mappings (weighted sum)

$$y = Wx + b$$



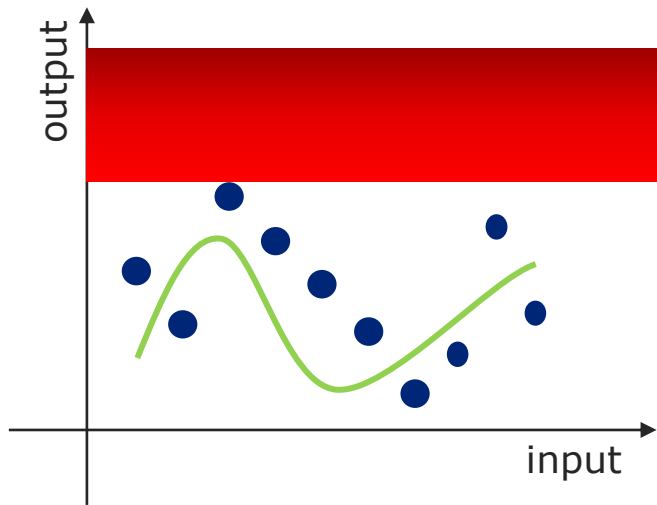
Nodes represent non-linear activation functions (ReLU, Sigmoid, ...)

$$\text{ReLU}(x) = \max(0, x)$$

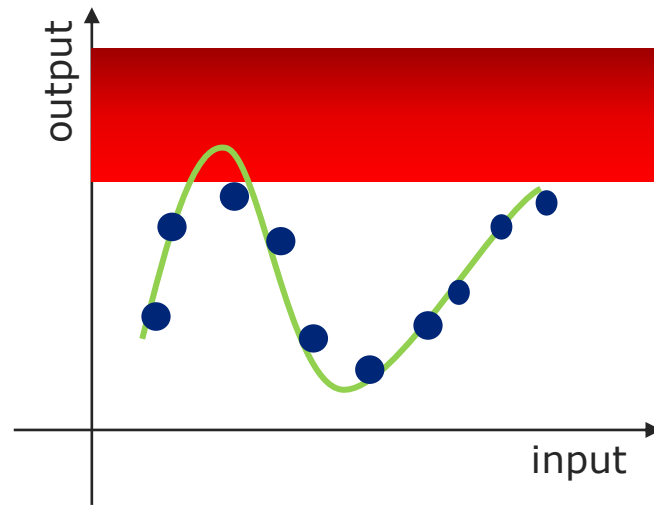
$$Y = f_4(f_3(f_2(f_1(X))))$$

# Context

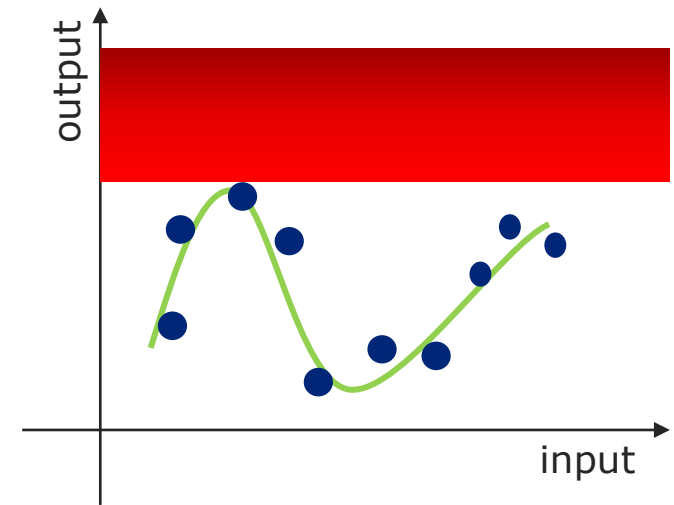
## NN Verification



Safe, but **not accurate**



Accurate, but **not safe**



Accurate **and** safe

— = Neural Network

● = Training sample

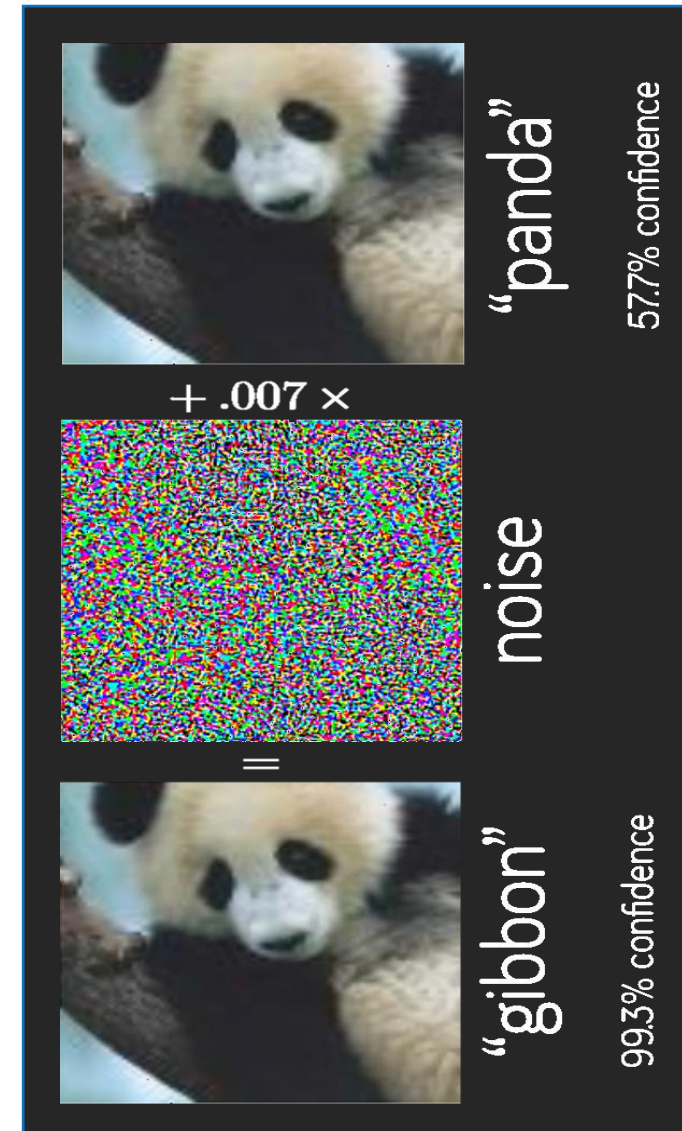
■ = Unsafe zone

# Context

## NN Verification

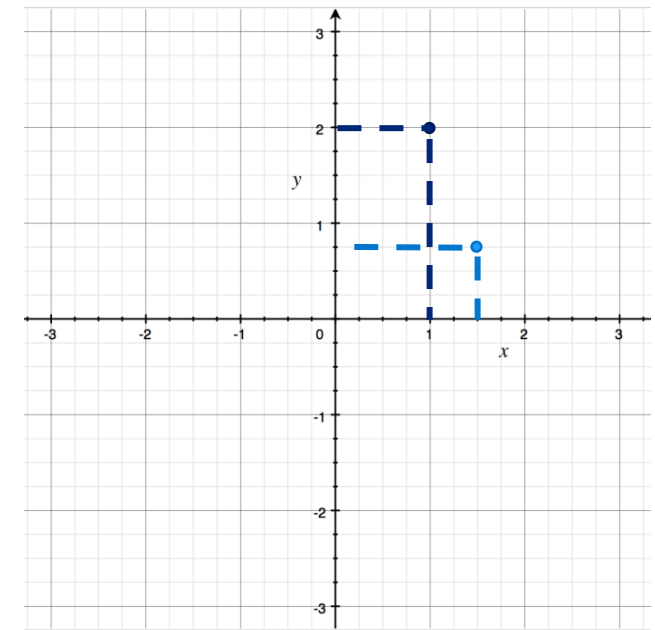
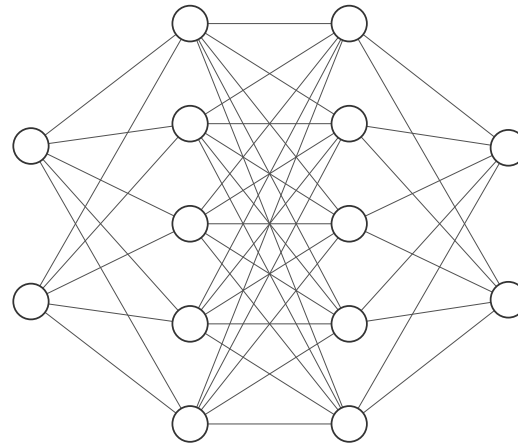
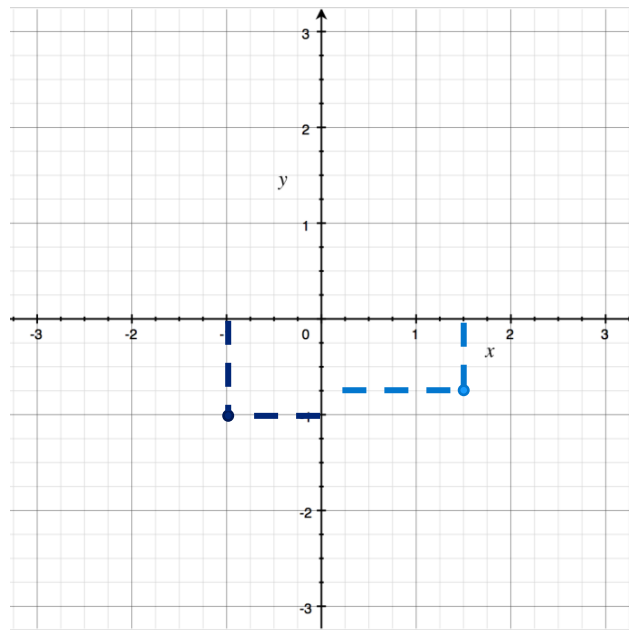
### Adversarial perturbation

- Minimal changes impact the classification
- Concerns for safety-critical applications
- Formal Verification for Input/Output specifications



# Abstraction algorithms

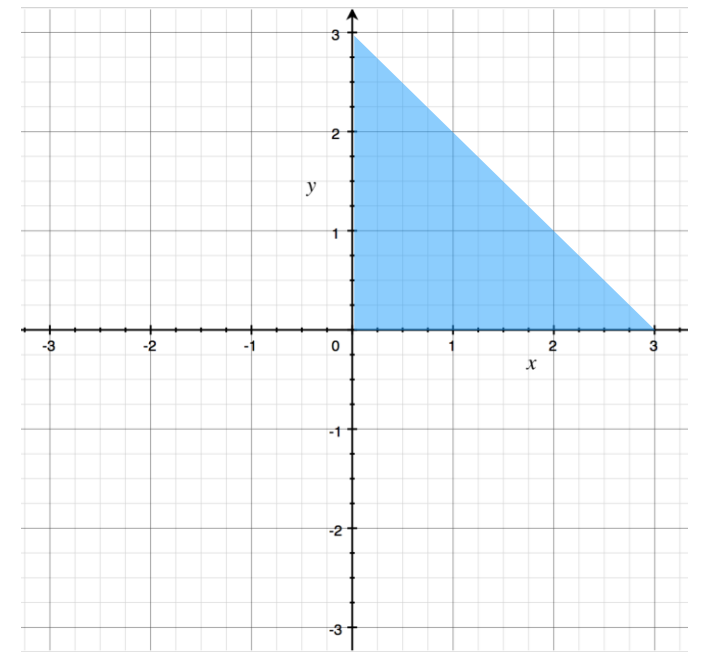
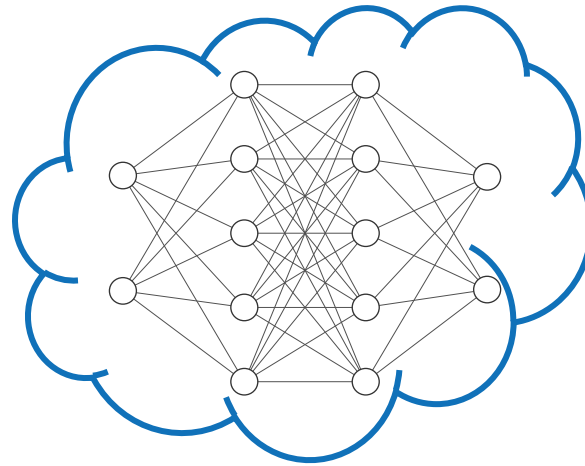
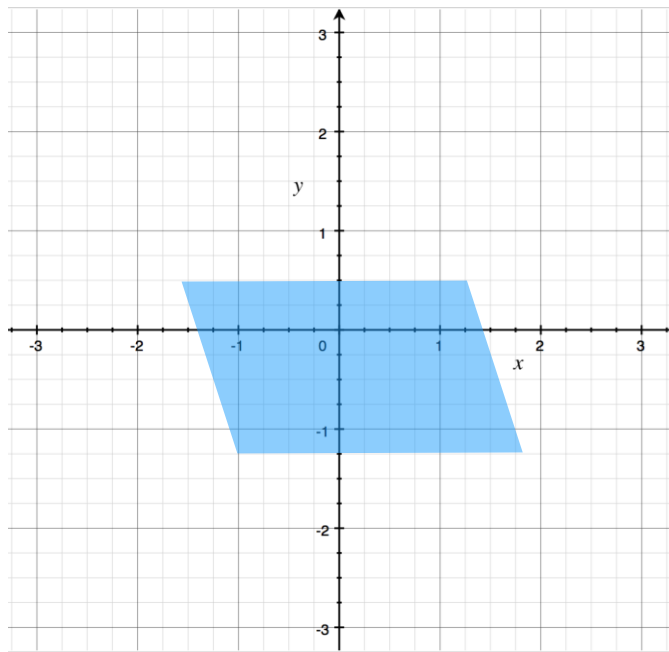
## Abstract Interpretation



Infeasible to run the NN on all inputs

# Abstraction algorithms

## Abstract Interpretation

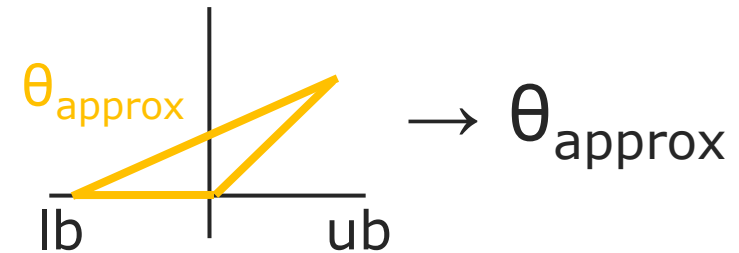
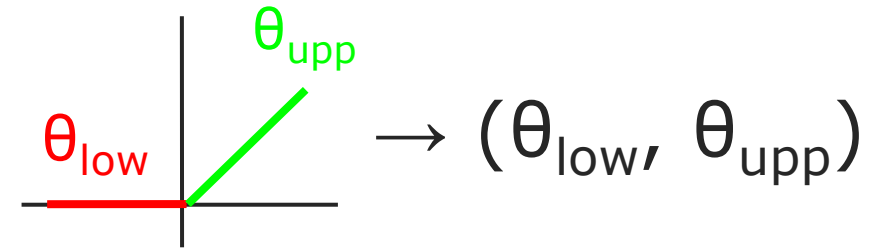
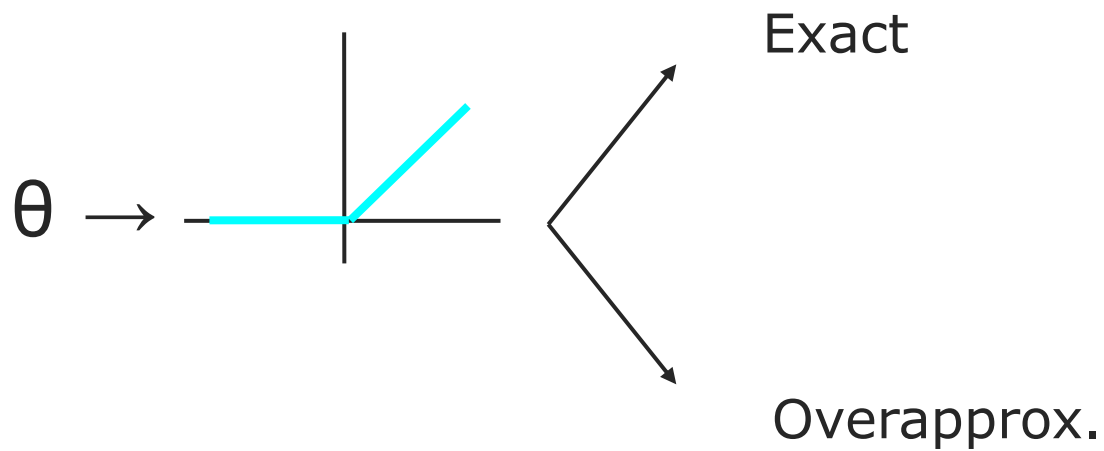


Abstraction provides finite approximation of (potentially) infinite sets



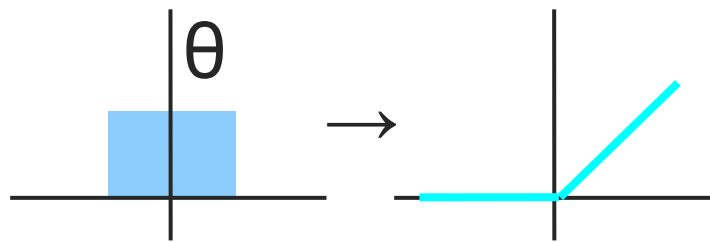
# Abstraction algorithms

## NN abstraction – ReLU layers



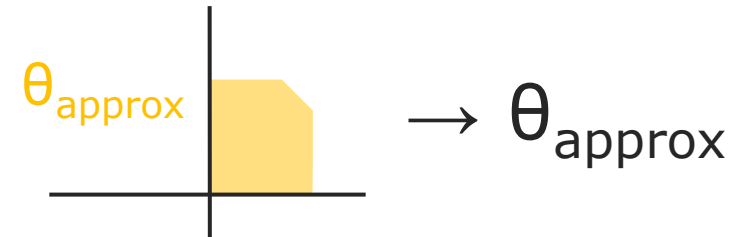
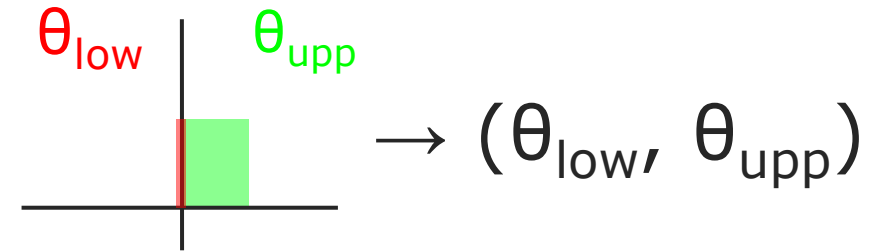
# Abstraction algorithms

## NN abstraction – ReLU layers



Exact

Overapprox.



# Abstraction algorithms

## Complete

Propagates the exact transformation of the input

If the input is **unstable** then we split

In the worst case the input set grows exponentially with the number of neurons

## Over-approximate

Propagates an approximation of the input

If the input is **stable** we keep the exact transformation

The approximation introduces a new variable and 3 constraints for each neuron

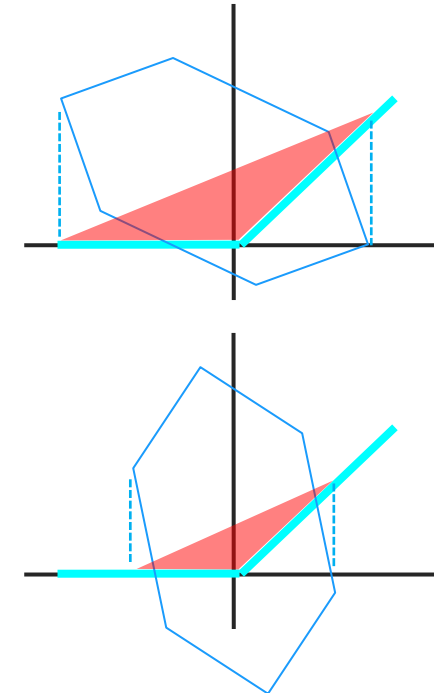
# Abstraction algorithms

## Mixed algorithm

The over-approximation abstract area depends on the set bounds

Approximate all neurons **but** the one with the greatest area

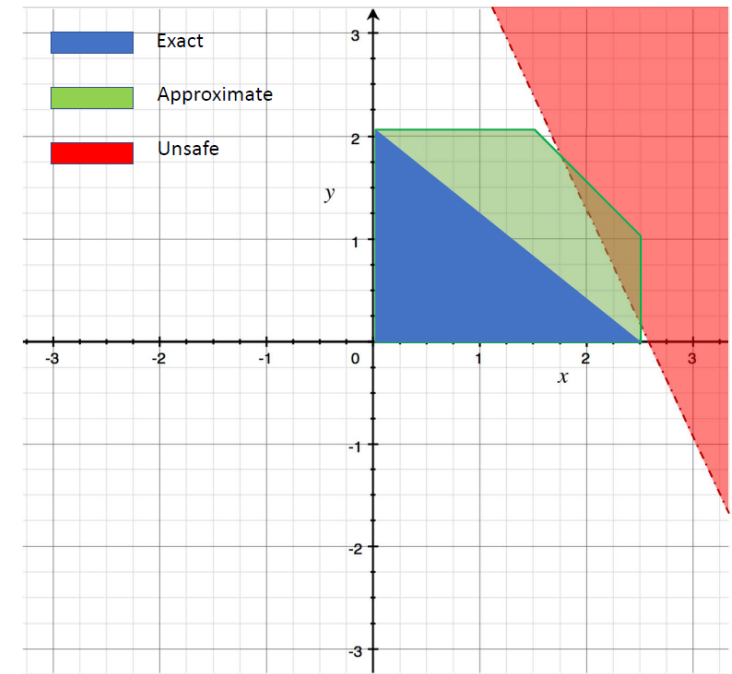
Still approximate, but faster and more precise



# CEGAR

## Approximation refinement

- If the exact output violates the safety property we can identify the unsafe input
- Not the same with the approximate output
- If we find a counterexample we can prove that the property is not verified



# CEGAR

## Approximation refinement

We enhance the refinement measuring **neuron relevance**<sup>1</sup>

Relevance is computed propagating the prediction backwards  
on samples from the output set

Measures the neurons contribution to the result

<sup>1</sup>Montavon, G. et al – Layer-Wise Relevance Propagation: An Overview; Explainable AI, 2019

# NeVer Tools

## A suite of tools for the manipulation and verification of NNs



pyNeVer – baseline API

CoCoNet – Tool for NNs manipulation and conversion

NeVer 2 – Tool for NNs learning and verification

*[neuralverification.org](http://neuralverification.org) | [github.com/NeVerTools](https://github.com/NeVerTools)*

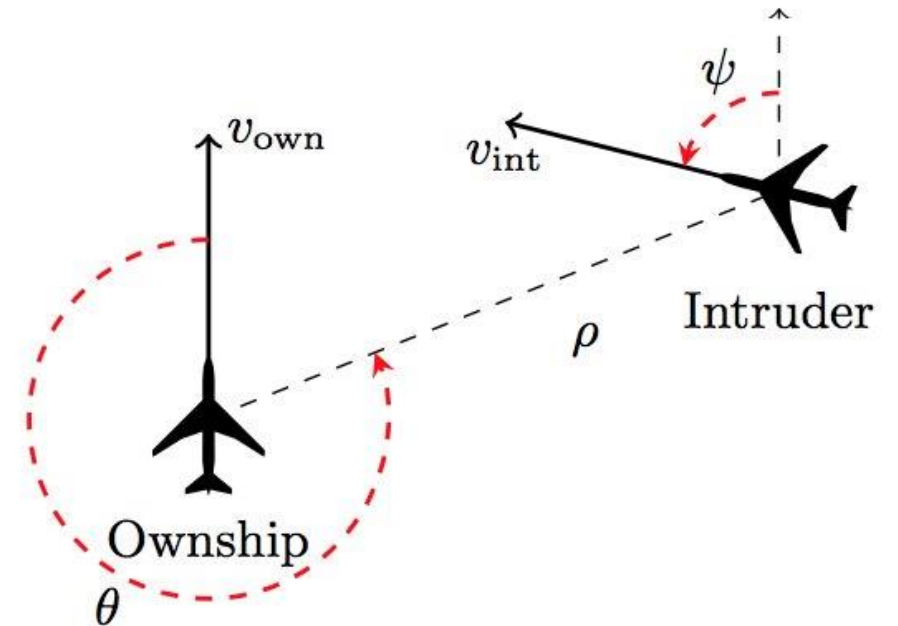
# Evaluation

## Verification of ACAS-Xu properties

Classic verification benchmark

Properties expressed for never issuing a  
Clear-Of-Conflict command

All properties are known to be verified





# Evaluation

PROPERTY	NETWORK	MIXED		CEGAR-PS		CEGAR-mR	
		TIME	VERIFIED	TIME	VERIFIED	TIME	VERIFIED
# 3	1_1	13	T	10	3/10	9	9/10
	1_3	10	T	14	6/10	10	0/10
	2_3	7	T	10	9/10	7	6/10
	4_3	15	T	17	10/10	14	10/10
	5_1	6	T	11	10/10	9	10/10
# 4	1_1	11	T	10	0/10	9	0/10
	1_3	8	T	16	0/10	11	0/10
	3_2	12	T	12	10/10	12	10/10
	4_2	12	T	11	10/10	12	10/10

# Remarks

## Explainability insights

We tried to enhance our refinement procedure

No clear improvement in results, but interesting insights

Unpredictability due to sampling

Working on better sampling/counterexample identification



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di Genova**